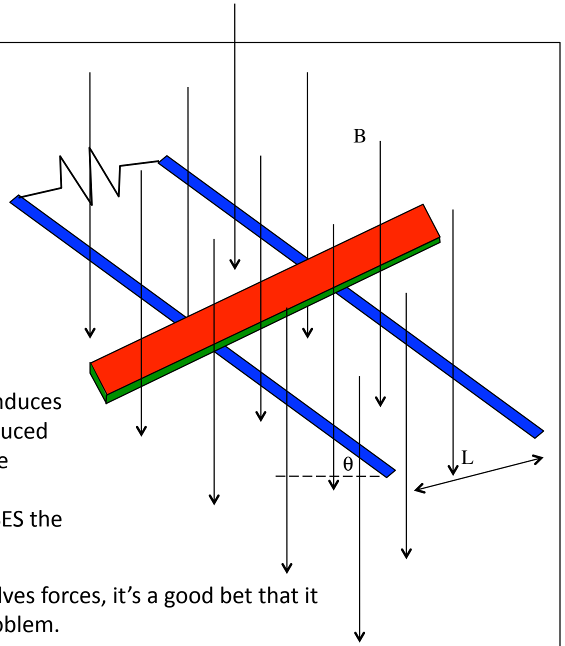
Problem 31.25

As the bar moves down the incline, the "coil" defined by the bar, the two rails and the resistor section has more and more magnetic field lines penetrating the face, which is to say the magnetic flux is increasing.



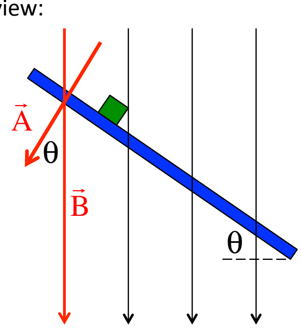
The changing magnetic flux induces an EMF that motivates an induced current that interacts with the external magnetic field and generates a force that OPPOSES the motion.

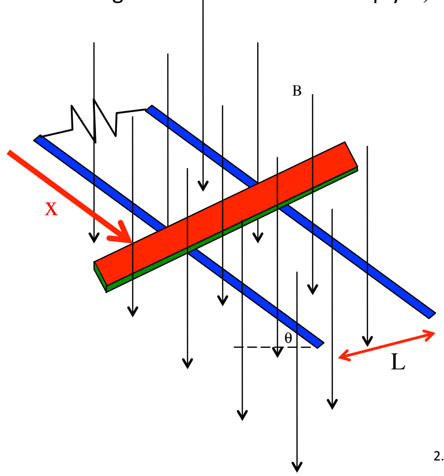
As this is a problem that involves forces, it's a good bet that it is a Newton's Second Law problem.

Additional parameters (x, for instance) are defined in the two sketches shown:

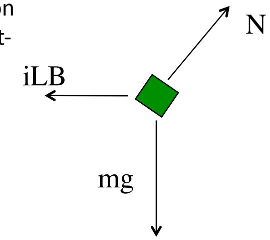
Note that the area-vector in $\Phi_B = \vec{B} \bullet \vec{A}$ has a magnitude of "Lx" and a direction that is perpendicular to the "face of the coil" (see sketch). (I could have made the area-vector direction upward, but there is no convention that requires that and the direction used makes the angle between $\vec{A}$ and $\vec{B}$ simply θ).

side view:





A free body diagram will help here, where the direction of induced magnetic force is defined by using the right-hand rule on $i\vec{L}x\vec{B}$. Using that, we get a force as shown.



So now for the pieces. We need to determine a general expression for the magnetic flux from which we can take a derivative and determine the induced EMF. Doing so yields:

$$\begin{aligned}\Phi_B &= BA\cos\phi \\ &= B(Lx)\cos\theta\end{aligned}$$

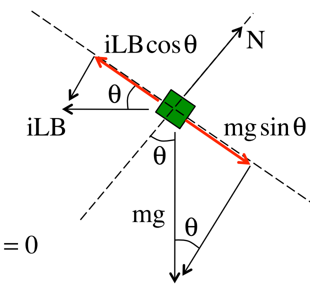
The induced EMF is:

$$\begin{aligned}\varepsilon &= -N\frac{d\Phi_B}{dt} \\ &= -(1)\frac{d\left(B(Lx)\cos\theta\right)}{dt} \\ &= -BL\cos\theta\frac{dx}{dt} \\ &= -BLv\cos\theta\end{aligned}$$

Using the magnitude of the EMF, we can get the induced current using Ohm's Law:

$$\begin{aligned}i &= \frac{\varepsilon}{R} \\ &= \frac{BLv\cos\theta}{R}\end{aligned}$$

We know the velocity down the incline is constant, so summing the forces in that direction yields:



$\sum F_{incline}$:

$$-iLB\cos\theta + mg\sin\theta = ma$$

$$\Rightarrow \quad -\left(\frac{BLv\cos\theta}{R}\right)LB\cos\theta + mg\sin\theta = 0$$

$$\Rightarrow \quad v = \frac{mgR\sin\theta}{L^2B^2(\cos\theta)^2}$$

With numbers:

$$
\begin{aligned}
v &= \frac{mgR\sin\theta}{L^2B^2(\cos\theta)^2} \\
&= \frac{(.2\ \text{kg})(9.8\ \text{m/s}^2)(1\ \Omega)(\sin\ 25^\circ)}{(1.2\ \text{m})^2(.5\ \text{T})^2(\cos\ 25^\circ)^2} \\
&= 2.8\ \text{m/s}
\end{aligned}
$$